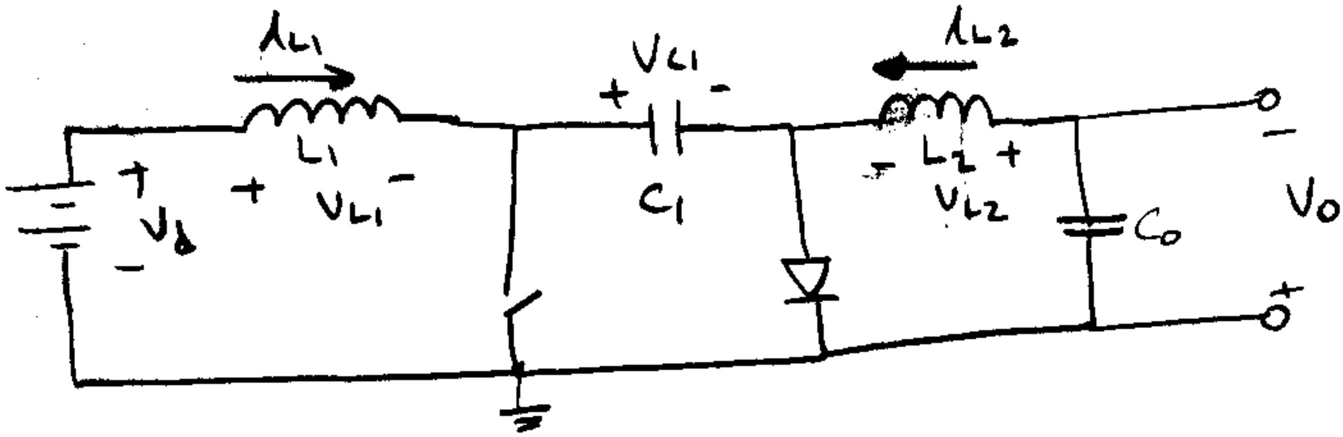
Cuk converter

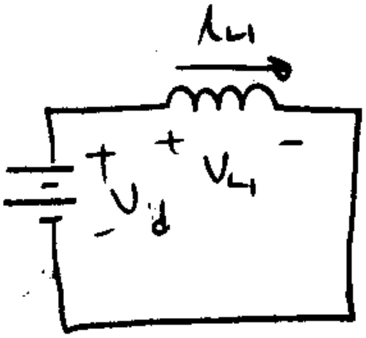


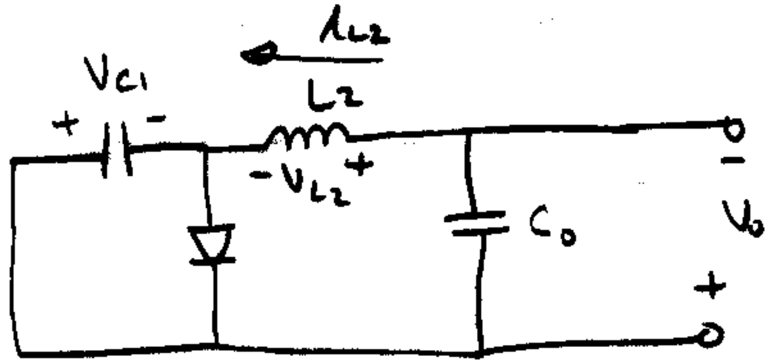
in steady state:

- The Avg voltage across $L_1 = 0$
- The Avg voltage across $L_2 = 0$

$\Rightarrow$ KVL $\Rightarrow$ $V_{C1} = V_d + V_o$

During ton: D = off





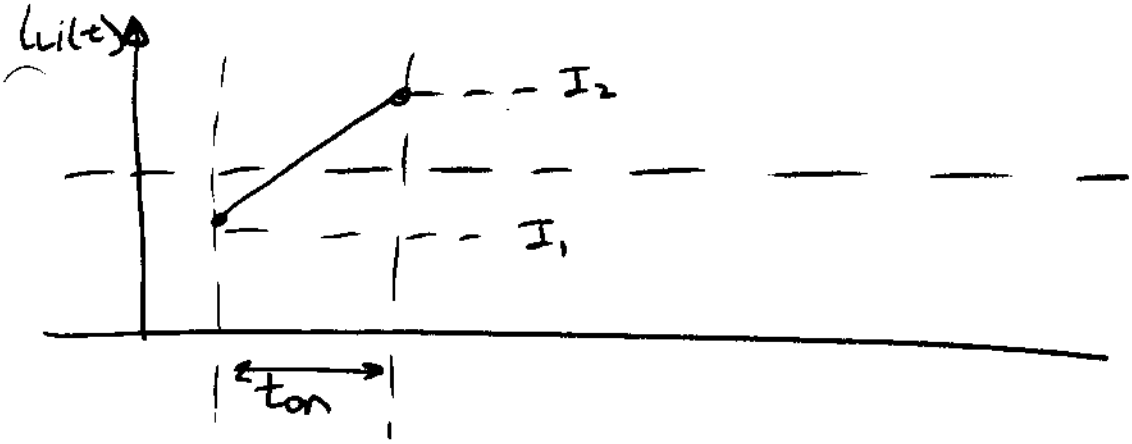
$V_{L1} = V_d$

i_{L1} will increase

$$V_{L2} = V_{C1} - V_o$$
$$= (V_d + V_o) - V_o = V_d$$

i_{L2} will increase

For L1

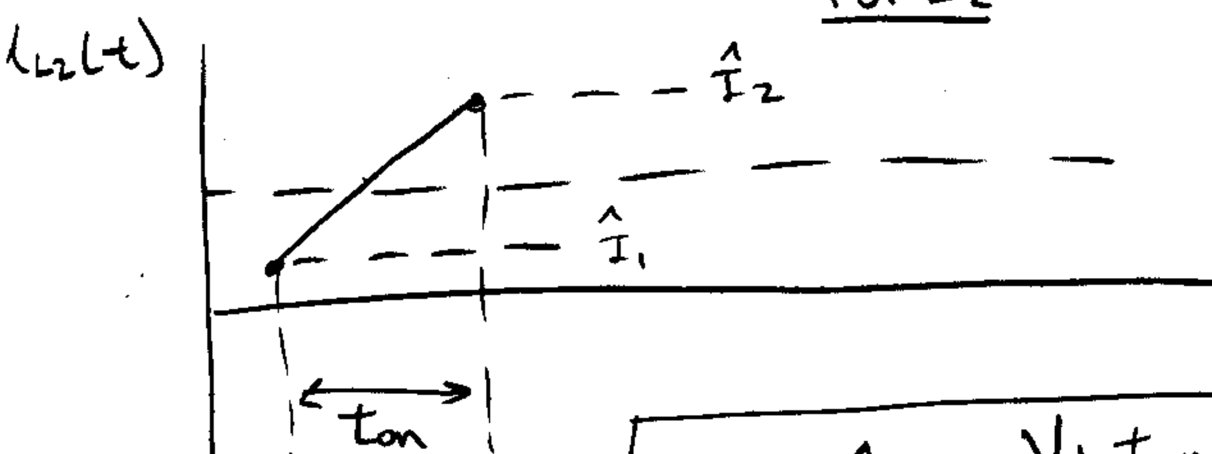


$$i_{L1}(t) = \int_0^{t_{on}} \frac{V_{L1}(t)}{L_1}\,dt + I_{C_0}$$

$$I_2 = \int_0^{t_{on}} \frac{V_d}{L_1}\,dt + I_C = \frac{V_d\,t_{on}}{L_1} + I_1$$

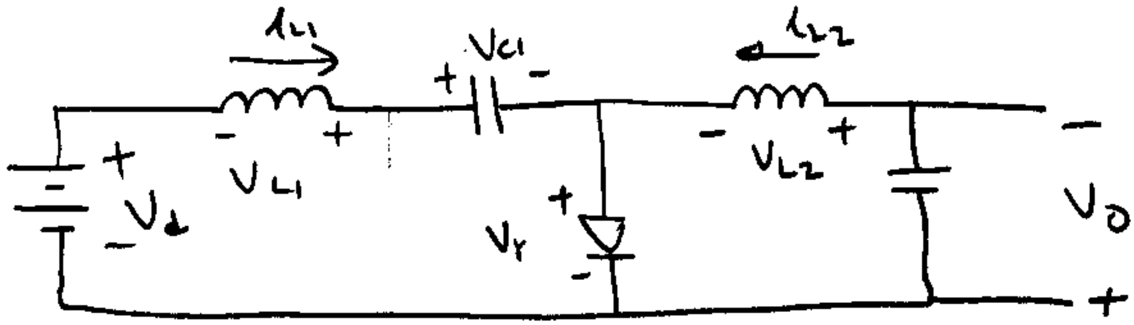
$$\boxed{I_2 - I_1 = \frac{V_d\,t_{on}}{L_1}} \quad (1)$$

For L2



$$\boxed{\hat{I}_2 - \hat{I}_1 = \frac{V_d\,t_{on}}{L_2}} \quad (2)$$

During t_{off}: Diode = on



$V_{L1} = V_{C1} - V_d =$

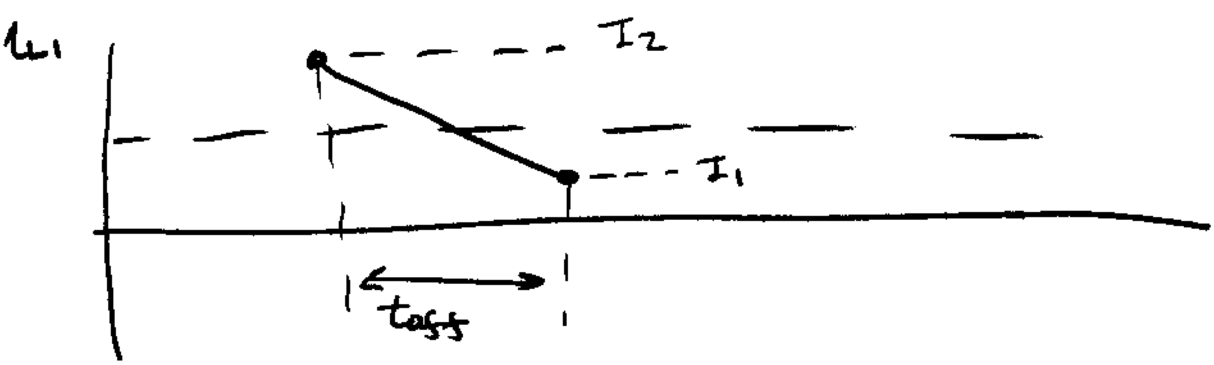
$= (V_d + V_o) - V_o$

$= -V_o$

$V_{L2} = -V_o$

i_{L2} decreases

i_{L1} decreases



$$i_{L1}(t) = \int \frac{V_{L1}(t)}{L} dt + I_C$$

$$I_1 = \int_0^{t_{off}} -\frac{V_o}{L_1} dt + I_2 = \frac{-V_o t_{off}}{L_1} + I_2$$

$$\Rightarrow \boxed{I_2 - I_1 = \frac{V_o t_{off}}{L_1}} \quad (3)$$

In a Similar manner

$$\hat{I}_2 - \hat{I}_1 = \frac{V_o t_{off}}{L_2} \quad (4)$$

Next, find Relationship between V_d + V_o

Set (1) + (3) equal to each other

$$\frac{V_d t_{on}}{L_1} = \frac{V_o t_{off}}{L_1} \Rightarrow \frac{V_o}{V_d} = \frac{t_{on}}{t_{off}}$$

but $t_{off} = T_s - t_{on}$

$$\frac{V_o}{V_d} = \frac{t_{on}}{T_s - t_{on}} = \frac{t_{on}/T_s}{(T_s - t_{on})/T_s} = \frac{D}{1-D}$$

$$\frac{V_o}{V_d} = \frac{D}{1-D} \quad (5)$$

ALSO, if $P_o = P_d$,

$$V_o I_o = V_d I_d$$

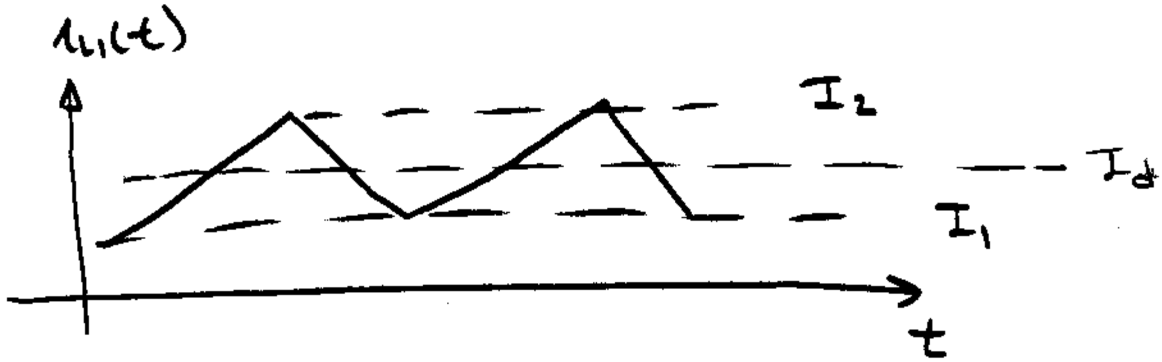
$$I_d = \frac{V_o}{V_d} I_o \Rightarrow \frac{I_d}{I_o} = \frac{D}{1-D} \quad (6)$$

Looking at the Circuit:

I_d = avg battery current

I_d = avg inductor current (L_1)

$$\left(\frac{I_1 + I_2}{2}\right) = I_d \quad (7)$$

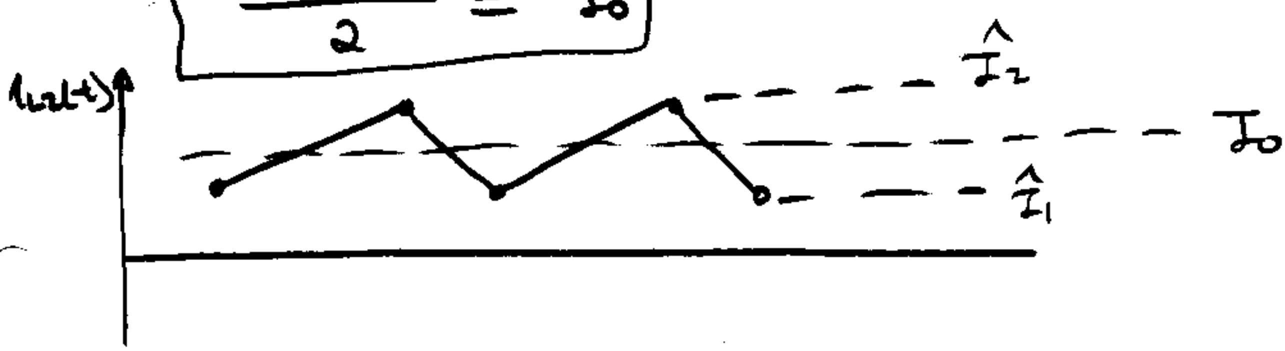


ALSO

I_o = average output current

I_o = avg inductor current (L_2)

$$\frac{\hat{I}_1 + \hat{I}_2}{2} = I_o \quad (8)$$



For L_1 current to be continuous, $I_1 > 0$

solve (7) & (1) for I_1

$$\left.\begin{aligned} I_2 - I_1 &= \frac{V_d t_{on}}{L_1} \\ \frac{I_1 + I_2}{2} &= I_D \end{aligned}\right\} \Rightarrow \boxed{L_1 > \frac{V_d t_{on}}{2 I_D}} \quad (9)$$

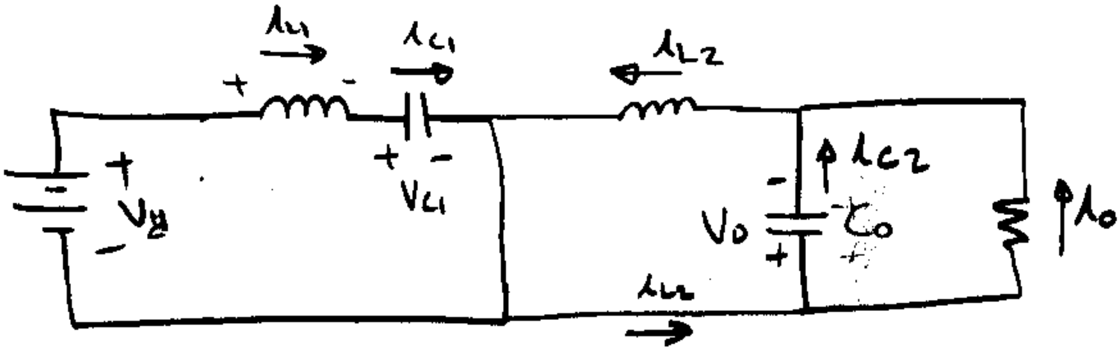
For L_2 current to be continuous, $\hat{I}_1 > 0$

solve (8) & (2) for $\hat{I}_1$

$$\left.\begin{aligned} \hat{I}_2 - \hat{I}_1 &= \frac{V_d t_{on}}{L_2} \\ \frac{I_1 + I_2}{2} &= I_o \end{aligned}\right\} \quad \boxed{L_2 > \frac{V_d t_{on}}{2 I_o}} \quad (10)$$

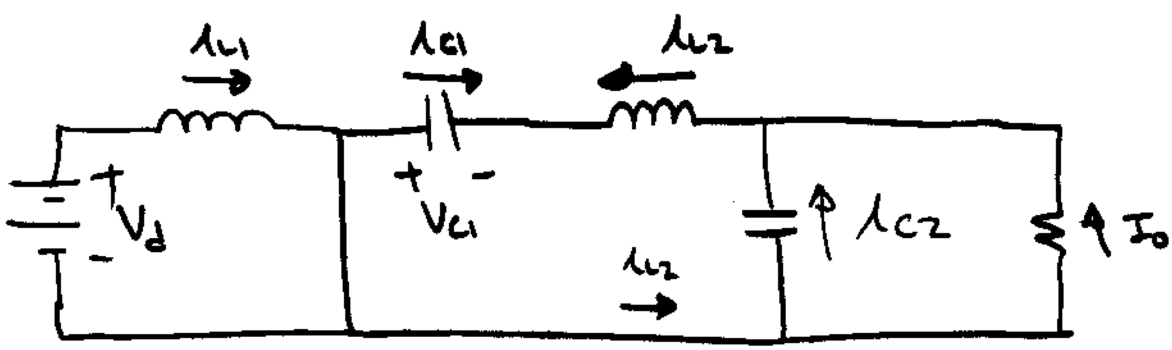
Now find the Capacitors

During t_{off} : D=on

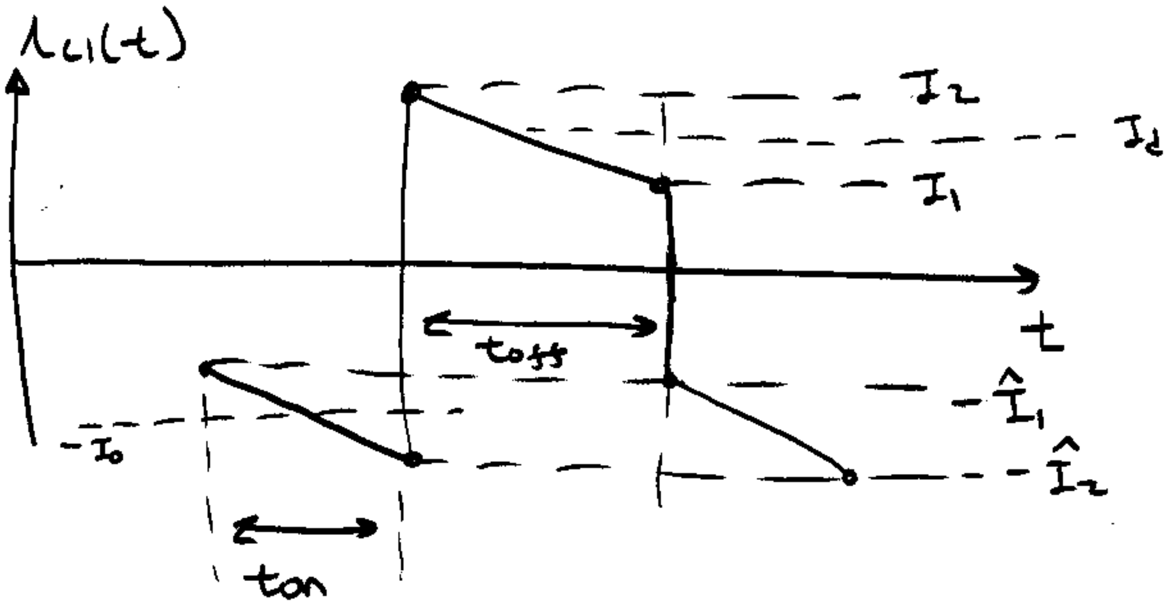


$i_{C1} = i_{L1}$ $\qquad$ $i_{C2} = i_{L2} - i_o$

During t_{on} : D=off



$i_{C1} = -i_{L2}$, $i_{C2} = i_{L2} - i_o$



For C_1 NO ESR

$$q_{1} = C_1 V_{C1} \Rightarrow \Delta q_{1} = C_1 \Delta V_{C1}$$

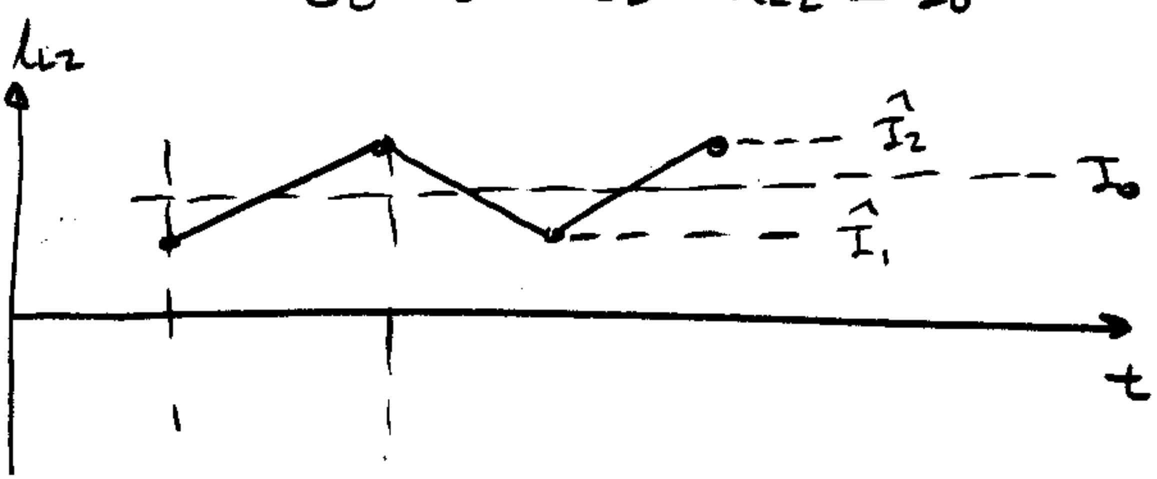
$$\Delta q_{1} = I_d t_{off} \quad \text{OR} \quad \Delta q_{1} = I_o t_{on}$$

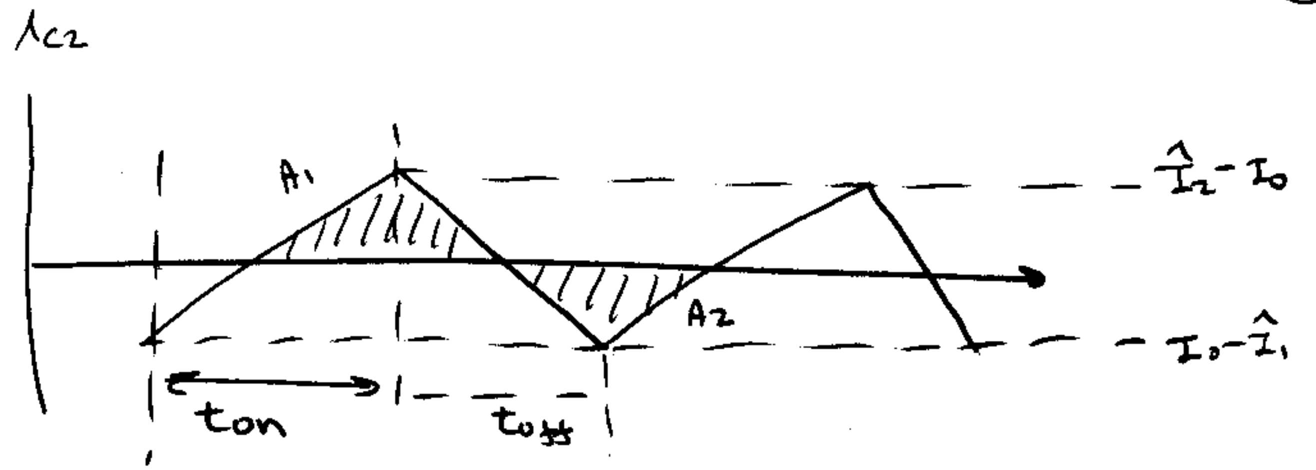
$$\Rightarrow \boxed{\Delta V_{C1} = \frac{I_d t_{off}}{C_1}} \quad \text{OR} \quad \Delta V_{C1} = \frac{I_o t_{on}}{C_1}$$

For C_1 with ESR

$$\Delta V_{C1}\Big|_{ESR} = \left(I_2 + \hat{I}_2\right) ESR\Big|_{C1}$$

C_o : $i_{C2} = i_{L2} - I_o$





Avg current during $A_1 \Rightarrow \dfrac{\hat{I}_2 - I_o}{2} = I_x$

$$I_x = \frac{\hat{I}_2 - \frac{\hat{I}_1 + \hat{I}_2}{2}}{2} = \frac{\hat{I}_2 - \hat{I}_1}{4}$$

I_x flows for $\frac{1}{2}$ period

$$\Delta q \Big|_{A_1} = \left(\frac{\hat{I}_2 - \hat{I}_1}{4}\right) \frac{T_s}{2} = \left(\frac{\hat{I}_2 - \hat{I}_1}{8}\right) T_s$$

For $C_o \Rightarrow \Delta q = C_o \Delta V_{co}$

$$\boxed{\Delta V_{co} = \left(\frac{\hat{I}_2 - \hat{I}_1}{8 C_o}\right) T_s}$$

From ESR: $\boxed{\Delta V_{co}\Big|_{ESR} = (\hat{I}_2 - \hat{I}_1)\, ESR\Big|_{co}}$

Cuk Converter Summary

$$\frac{V_o}{V_d} = \frac{D}{1-D} \quad (7), \qquad \frac{I_o}{I_d} = \frac{1-D}{D} \quad (8), \qquad D = \frac{t_{on}}{T_s}$$

For L_1

$$I_2 - I_1 = \frac{V_d \, t_{on}}{L_1} \quad (1)$$

$$\frac{I_1 + I_2}{2} = I_o \quad (2)$$

$$L_1 > \frac{V_d \, t_{on}}{2 I_d} \quad (3)$$

For L_2

$$\hat{I}_2 - \hat{I}_1 = \frac{V_d \, t_{on}}{L_2} \quad (4)$$

$$\frac{\hat{I}_2 + \hat{I}_1}{2} = I_o \quad (5)$$

$$L_2 > \frac{V_d \, t_{on}}{2 I_o} \quad (6)$$

$$\Delta V_{C1}\Big|_{\substack{NO\\ESR}} = \frac{I_d\, t_{off}}{C_1} \quad \text{OR} \quad \Delta V_{C1}\Big|_{\substack{NO\\ESR}} = \frac{I_o\, t_{on}}{C_1}$$

$$\Delta V_{C1}\Big|_{ESR} = \left(I_2 + \hat{I}_2\right) ESR\Big|_{C_1} \qquad (9)$$

$$\Delta V_{C_o}\Big|_{\substack{NO\\ESR}} = \left(\frac{\hat{I}_2 - \hat{I}_1}{8C}\right) T_s \qquad \Delta V_{C_o}\Big|_{ESR} = \left(\hat{I}_2 - \hat{I}_1\right) ESR\Big|_{C_o}$$

(10)